hydrophilic exterior of lysozyme unfolds more readily than the hydrophobic core in urea at pH 2.8. The action of ethanol-water on trypsin has been interpreted by POHL[41] as a two state process (cf.[42]).

Following the classic paper of MIRSKY and PAULING[43] the denaturation of proteins was interpreted largely in terms of rupture of peptide hydrogen bonds. However, more recently it has become fashionable to lay great stress on the importance of the hydrophobic bond and the large heat capacity changes for denaturation reactions[12,13]. It is difficult to reconcile the hydrophobic interpretation with the temperature dependence of 'heat' denaturation in general, and with the effect of pressure on the model compound[44] 4-octanone, and on ribonuclease[45]. KAUZMANN and KLIMAN[44] found that the solubility of 4-octanone in water, at constant temperature, increases with increasing pressure up to 1500 to 2500 kg/cm² when it is ca. 1.5 times the value at 1 atm. Thereafter it decreases becoming equal to the value at 1 atm when the pressure is 4200, 5300 and 6500 kg/cm² at 15, 25 and 35 °C respectively. They found the solubility at 15 °C is greater than that at 35 °C for pressures up to 4500 kg/cm². The volume change on solution is ca. —28 cm³/mole at 1 atm (15–35 °C), 0 cm³/mole at 2000 kg/cm², and +3 to +5 cm³/mole at pressures above 2000 kg/cm². It was concluded that 4-octanone is a good compound to use as a model for hydrophobic interactions. On the basis of this model it would be expected that proteins would have a greater tendency to denature at pressures up to 4500 kg/cm² at 15 °C and up to 6000 kg/cm² at 35 °C, than at 1 atm. Assuming the solubility trends for 4-octanone continue at higher pressures native protein conformations should be stabilized at such pressures. On the other hand, as KAUZMANN and KLIMAN stress, pressures below 3000–4000 kg/cm² do not denature many proteins. Indeed the native conformations, except that for ribonuclease, appear to be stabilized by these pressures. When the pressure is increased above 4000–6000 kg/cm² at 10–70 °C all proteins and enzymes studied to date are denatured. The volume change of —45 cm³/mole observed by BRANDTS et al.[45] for the conformational change for ribonuclease in water at 25 °C is much smaller than one would expect if hydrophobic bonding is a major structural force in ribonuclease. The contribution of the hydrophobic bond in protein denaturation will be reviewed elsewhere by H. McK., who concludes that in many cases side chain hydrogen bonds are the more decisive factor[46].

Since this manuscript was prepared a review on microcalorimetry by I. WADSÖ has appeared in Q. Rev. Biophys. *3*, 383 (1970). WADSÖ stresses that heat capacity determinations on protein solutions are quite difficult to perform and that, although the required precision can be achieved with the best isoperibol type calorimeters, large quantities of protein (ca. 100 ml, 1% solution) are required.

Zusammenfassung. Untersuchungen der Denaturierungsvorgänge sind für ein Verständnis der Proteinstruktur und -funktion von grosser Bedeutung. Es werden Methoden besprochen, die am ehesten geeignet sind, die Fragen zu beleuchten, ob die Denaturierung einen Zwei-Formen-Prozess darstellt und ob sie reversibel oder irreversibel ist. Sie werden an Beispielen der Harnstoffdenaturierung genetischer Varianten des Rinder-β-Lactoglobulins erläutert. Es wird ausserdem erörtert, in wieweit sich die Schlussfolgerungen auf die Denaturierung anderer Proteine übertragen lassen.

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SPECIALIA

A New Solution of the Equilibrium Equation for an Isothermal Gas Sphere

In our recent paper[1] we have extended the region of validity of the Fowler's solution for the Lane-Emden equation of index 3. In this paper we have obtained a new solution of the equilibrium equation in the (ξ, ψ) plane for an isothermal gas sphere, satisfying the required boundary conditions and asymptotically approaching the singular solution at infinity. The solution governs the density distribution at and around the centre.

The equilibrium equation for an isothermal gas sphere is[2]

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{d\psi}{d\xi}\right] = e^{-\psi}, \quad (1)$$

where ξ and ψ are functions of radius and density given by

$$\xi = \alpha r, \quad \varrho = \lambda e^{-\psi}.$$

Equation (1) is analogous to the Lane-Emden equation. The solution of (1) is valuable if it satisfies the following boundary conditions:

$$\psi = 0, \quad \frac{d\psi}{d\xi} = 0, \quad \text{at} \quad \xi = 0. \quad (2)$$

With the help of Emden's transformation and the homology theorem, (1) can be written in the following form:

$$y\frac{dy}{dz} - y + e^z - 2 = 0, \quad (3)$$

where

$$y = \frac{dz}{dt}, \quad z = -\psi + 2\log\xi, \quad e^{-t} = \xi. \quad (4)$$

By an application of HARDY's[3] theorem to (3), it follows that we should ultimately have one of the following three possibilities:

$$\frac{dy}{dz} \to 0, \quad \frac{dy}{dz} \to \infty, \quad \frac{dy}{dz} \to 1 \quad \text{as} \quad \xi \to 0.$$

The first possibility yields E-curve. The second possibility is clearly impossible. Under the third possibility CHANDRASHEKHAR[2] found the solution of (1) in the form $\varrho = A\,\xi^{-2}\,e^{-C/\xi}$ at $\xi \to 0$, where A and C are constants. Now we seek a solution valid in a larger region $0 \le dy/dz < 1$ at and around the centre $\xi = 0$. Equation (3) can be written as

$$\frac{y}{e^z - 2} = 1 + \left(\frac{dy}{dz}\right) + \left(\frac{dy}{dz}\right)^2 + \ldots + \left(\frac{dy}{dz}\right)^n + \ldots \quad (5)$$

We choose a sufficiently large positive integer m, such that for $n > m$, $(dy/dz)^n \to 0$ in the region $0 \le dy/dz < 1$. Thus equation (5) approximates to the form:

$$\frac{y}{e^z - 2} = \left[1 - \left(\frac{dy}{dz}\right)^{m+1}\right]\left(1 - \frac{dy}{dz}\right)^{-1}. \quad (6)$$

Substituting for dy/dz from (3) into (6) we get

$$[y - (e^z - 2)]^{m+1} = \varepsilon^{m+1} \approx 0, \quad \text{for} \quad 0 \le \varepsilon < 1, \quad (6\text{a})$$

which yields the following family of solutions in the (ξ, ψ) plane:

$$B\,\xi^\varepsilon = \xi^2 - (2 - \varepsilon)\,e^{-\psi}, \quad (7)$$

where B is the parameter of the family. To get a particular solution satisfying the boundary conditions (2), we see that $\varepsilon = 0$ and $B = -2$. The corresponding solution is called the E-solution and is given by

$$e^{-\psi} = 2\,(\xi^2 + 2)^{-1}. \quad (8)$$

The structure of the complete isothermal gas sphere can be determined when a solution of (1) satisfying the boundary conditions (2) can be obtained. Therefore, the E-solutions are significant, as they form a grid for use in analyzing the other solutions. According to RUSSEL[4] only E-solutions can represent independent masses of fluid throughout the extent, but others may hold for a mass of fluid exterior to a sphere of suitable mass and radius. Thus the family of solutions (7) serves as an internal support.

CHANDRASHEKHAR[2] states that 'the solutions of the isothermal equation, which are finite at origin, have necessarily $d\psi/d\xi = 0$ at $\xi = 0$; and that consequently the homologous family $\{\psi(\xi)\}$ includes all solutions which are finite at the origin'. For the verification of this statement we can write equation (1) in the following form:

$$\frac{d^2x}{d\xi^2} = \xi\,e^{-x/\xi}, \quad (9)$$

where

$$x = \psi\,\xi \quad \text{or} \quad e^{-\psi} = e^{-x/\xi}. \quad (10)$$

From (10) we have

$$\left(\frac{d\psi}{d\xi}\right)_{\xi=0} = \lim_{\xi=0}\left[\frac{\xi\,dx/d\xi - x}{\xi^2}\right]. \quad (11)$$

For any solution passing through $x = 0$, $\xi = 0$, we have

$$x(\xi) = \xi\left(\frac{dy}{d\xi}\right)_{\xi=0} + \frac{\xi^2}{2}\left(\frac{d^2x}{d\xi^2}\right)_{\xi=0} + \ldots \quad (12)$$

$$\frac{dx}{d\xi} = \left(\frac{dx}{d\xi}\right)_{\xi=0} + \xi\left(\frac{d^2x}{d\xi^2}\right)_{\xi=0} + \ldots \quad (13)$$

Now from equations (11), (12) and (13) we get

$$\left(\frac{d\psi}{d\xi}\right)_{\xi=0} = \frac{1}{2}\left(\frac{d^2x}{d\xi^2}\right)_{\xi=0} = \frac{1}{2}\lim_{\xi=0}\,[\xi\,e^{-x/\xi}]. \quad (14)$$

Substituting the value of $e^{-x/\xi}$ from (8) into (14) we get

$$\left(\frac{d\psi}{d\xi}\right)_{\xi=0} = \frac{1}{2}\lim_{\xi=0}\left[\frac{2\,\xi}{\xi^2 + 2}\right] = 0.$$

Thus the E-solution (8) varifies the statement of CHANDRASHEKHAR. The singular solution of (1) obtained by ZÖLLNER[5] is $e^{-\psi} = 2/\xi^2$. Hence the solutions (7) and (8) asymptotically approach the singular solution at infinity.

Zusammenfassung. Eine neue Lösung der Gleichgewichtsgleichung in der (ξ, ψ)-Ebene wurde für eine isotherme Gaskugel gefunden.

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